

Learning and Assortative Matching with Private Histories

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Abstract

The paper models an economy in which output is produced by teams of size two, match partners' qualities are complements, and agents' histories are private information. The initial situation is a random assignment caused by a technological change that alters agents' productive skills. Agents must relearn their types by observing the output that they and their match partners produce. The economy adjusts toward its long-run positive assignment at a geometric rate when separating output-contingent contracts are available. When contracts fail and output is split equally, adverse selection slows adjustment to an arithmetic rate and, for some parameter values, stops it altogether. Along the transition path, the middle output state shrinks and mass moves to the extremes. Thus the speed of convergence to assortative matching depends on whether firms can commit to contracts that screen workers by their private histories.

Keywords: Sorting, Assortative Matching, Adverse Selection, Private Histories

JEL Codes: D82, J24, J31, O33

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1 Introduction

This paper studies the adjustment path after a technology shock that “garbles” workers’ productivities. Output is produced in two-worker teams, qualities are complements, and only team output is observed. Agents learn about their own types from realized match output, but output histories are private. Firms post contracts and compete for workers. The central question is how private histories affect the speed of reallocation toward the positive assignment, in which each high type works with another high type (Becker 1973; Shimer and Smith 2000).

The transition path is characterized in closed form and three results follow. First, output-contingent contracts screen workers whose types are already known away from workers whose types are unresolved. Second, the availability of separating contracts determines the speed of adjustment. With contracts, the convergence rate is pinned down by the information structure alone. The economy converges geometrically to the positive assignment, with the share of middle-output teams halving each period at the same rate under every separating contract. When contracts fail and output is split equally, known low types re-enter the meeting pool and create adverse selection. This slows convergence to an arithmetic rate and, for low enough productivity gains, a small enough share of high types, and sufficiently impatient agents, stops adjustment altogether. Third, the sorting process moves mass out of the middle output state and into low- and high-output teams.

The main results rest on complementarity in production and on private histories. The model has two types and a general strictly supermodular production function. This provides a parsimonious setting in which team output reveals that a match is mixed without identifying which worker is high. The two-type structure also gives closed-form laws of motion and makes the difference between geometric and arithmetic adjustment transparent. The model is therefore an example meant to isolate this source of slow adjustment, rather than to provide a general theory of sorting under private information.

The model does not explain why productivities were disrupted in the first place.¹

¹Andolfatto and Smith (2001, p. 743) would refer to such a disruption as a paradigm shift:

Instead, it takes that disruption as given and asks how long it takes the economy to sort back toward the positive assignment. Whether that process runs fast or slow depends not on the size of the original shock, but on whether firms are able to use contracts that screen out workers who have already learned their types.

Kremer and Maskin (1996) show that skill-biased technological change produces segregation and rising wage inequality. This paper instead focuses on the adjustment path after a shock disrupts existing matches.

In contrast to Anderson and Smith (2010) and Doppelt (2016), where reputations evolve publicly, histories here are private. Related models with adverse selection include Guerrieri and Shimer (2014) and, in a static competitive search setting, Guerrieri, Shimer, and Wright (2010). Kahn (2013) documents asymmetric information between employers in the labor market, consistent with the private-history setting studied here. Kyle (1985) and Golosov, Lorenzoni, and Tsyvinski (2014) provide other settings in which private information slows adjustment.² A similar slowdown also appears in Wolinsky (1990). The difference here is that the shock garbles abilities so agents must relearn their types through team output, and the paper studies the aggregate transition to the positive assignment. Smith (1992) also obtains arithmetic convergence from a different source, while Ljungqvist and Sargent (1998) show protracted adjustment after a shock. The present model differs from all of these in combining endogenous learning about own type with aggregate dynamics.

Section 2 presents the model. Section 3 studies separating contracts. Section 4.1 gives the public-history benchmark, and Section 4.2 discusses the Anderson-Smith directed-learning assignment. Section 5 studies equilibrium dynamics when contracts fail and private histories interact with equal splitting. Section 6 gives the distributional implication using the increasing convex order. Section 7 concludes.

¹“Assume that individuals attach zero probability to the event of a paradigm shift. This behaviour is approximately optimal if paradigm shifts occur with sufficiently low probability. Conditional on a paradigm shift, however, aggregate dynamics will follow a perfect foresight path.”

²Adverse selection on one side of the market, without agents learning their own types, is analyzed by Greenwald (1986), Inderst (2005), and Kugler and Saint-Paul (2004).

2 Model

There is a unit continuum of infinitely lived, risk-neutral agents with discount factor $\beta \in (0, 1)$. Each match has two workers. A worker is either low (L) or high (H), and ϕ is the fraction of high types. If f denotes output per worker as a function of the two workers' types, we assume $f(H, H) = A$, $f(H, L) = f(L, H) = B$, and $f(L, L) = C$. Equivalently, output per worker is

$$y = \begin{cases} A & \text{if both workers are high} \\ B & \text{if one worker is high and one is low} \\ C & \text{if both workers are low} \end{cases}, \quad (1)$$

where

$$A > B > C > 0, \quad A + C > 2B. \quad (2)$$

The second inequality is strict supermodularity.³ It is the two-type version of the complementarity condition that drives positive assortative matching (Becker 1973). For example, suppose worker quality is $x \in \{1, r\}$ with $r > 1$, and output per worker is $y = x_1 x_2$. Then $C = 1$, $B = r$, and $A = r^2$, and strict supermodularity is

$$r^2 + 1 > 2r,$$

or equivalently $(r - 1)^2 > 0$, as required by (2).

Time is discrete and output is produced at the end of the period. The parties to the match may remain together or separate. Matching is instantaneous and no one is unemployed.

In period zero matches have already formed randomly and at the outset agents do not know their abilities. But y is observed without error,⁴ and so outputs $y \in \{C, A\}$

³With two types, strict supermodularity means $f(H, H) + f(L, L) > f(H, L) + f(L, H)$. Since $f(H, L) = f(L, H) = B$, this is exactly $A + C > 2B$. For a differentiable production function, the analogous condition is a positive cross-partial derivative, $\partial^2 f / \partial x_1 \partial x_2 > 0$.

⁴With noisy signals, non-middle outcomes would move beliefs away from the endpoints rather than revealing types fully. In a simple noisy analogue, each worker's signal equals her type with probability $\alpha \in (1/2, 1]$, and observed output is A , B , or C according to the two signals. If a worker

reveal both workers' types perfectly. On the other hand, output $y = B$ reveals that the match contains one high type and one low type, but it does not identify which worker is high. It therefore leads to the end-of-period-zero belief

$$\pi = \frac{(1 - \phi) \phi}{\phi(1 - \phi) + (1 - \phi) \phi} = \frac{1}{2}.$$

Thus at the end of period zero there are three agent types indexed by the belief $\pi \in \{0, \frac{1}{2}, 1\}$ of being a high type. The paper's central question is how the economy moves from this random assignment toward the positive assignment.

An equilibrium or benchmark specifies matching and separation decisions, wages or sharing rules, and beliefs that satisfy Bayes rule whenever possible. In the contract case, firms post output-contingent wages and workers decide whether to apply. In the no-contract case, partners split output equally and decide whether to stay together or separate. Agents who separate are rematched instantly.

An agent's strategy is a mapping from his private output history to an action: whether to apply, stay, or separate, and on what terms. A Bayes-Nash equilibrium consists of a strategy profile and a belief system over types given private histories such that each agent's strategy maximizes his expected discounted lifetime income given his beliefs and the equilibrium strategies of others, and beliefs follow from Bayes rule given equilibrium play. In the no-contract case, we refine this equilibrium concept by selecting the maximal-duration recombination equilibrium defined in Section 5.

3 Firms post vacancies and contracts

This section analyzes risk-neutral firms that post vacancies with output-contingent contracts. They choose a wage schedule $w(y)$ to maximize expected profit, taking workers' strategies as given. A zero-profit condition then restricts the equilibrium wage schedule. The zero-profit condition is an *ex-ante* condition: Before output is

has belief $\pi = 1/2$, the probability of a high signal is $\alpha/2 + (1 - \alpha)/2 = 1/2$. Hence two such workers produce the middle signal with probability 1/2. We keep deterministic output signals so the posterior state is simply $\{0, 1/2, 1\}$.

realized, the expected profit of the posting firm is zero.

Agents' histories are private, and firms offer contracts to which they are able to commit. Two applicants are chosen and their payments are contingent on the output that their match produces. A contract cannot condition on an agent's history because histories are private. Instead, a firm will design its contract to attract only those types that it wants to hire. The contracts will attract agents with beliefs $\pi = 1/2$ and screen out agents with $\pi \in \{0, 1\}$. Denote the wage by w_y for $y \in \{C, B, A\}$. The match then remains in place forever if $y \in \{C, A\}$ and workers are paid y thereafter. But if $y = B$, the workers are paid w_B and are then fired and must seek a new match. Firing here is a commitment device. Since team output $y = B$ does not identify which worker is the high type, retaining the pair would leave both workers' types unresolved. Dissolving the match and returning both workers to the applicant pool allows sorting to continue. These terms are understood by all applicants.

All applications will then come from workers who have produced $y = B$ since the beginning and therefore have belief $\pi = 1/2$. In that equilibrium, taking a job will, for any type π , mean that one's coworker is equally likely to be of a high or a low type. The equilibrium will thus be a separating equilibrium: Agents sort by their private belief π , so that only those with $\pi = 1/2$ participate in this market. Within any given match one worker may turn out to be a high type and the other a low type, but this is revealed only by the output realization, not before. The separation is over beliefs, not over types per se.

An equilibrium contract must satisfy four conditions. If only the $y_{-1} = B$ agents apply (where y_{-1} is the previous period output of an individual), then half of the applicants are high types and half are low types. A firm that posts a vacancy will offer the contingent contract w_y that

1. *Induces all workers in $y_{-1} = B$ matches whose beliefs are $\pi = 1/2$ to abandon their matches and apply.*—Given that one's coworker also has $\pi = 1/2$, the outputs $y \in \{C, B, A\}$ have probabilities $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ respectively. An output $y = B$ leads to a separation, whereas $y = C$ and $y = A$ lead to continuation values of $\beta C/(1 - \beta)$ and $\beta A/(1 - \beta)$ respectively. Let W be a $\pi = 1/2$ type's lifetime value associated with

applying. Then W must satisfy the Bellman equation and inequality

$$W = \frac{w_C + \frac{\beta C}{1-\beta}}{4} + \frac{w_A + \frac{\beta A}{1-\beta}}{4} + \frac{w_B + \beta W}{2} \geq \frac{B}{1-\beta}. \quad (3)$$

2. *Deters the low ($\pi = 0$) types from applying.*—Knowing that he is a low type, someone coming from a $y_{-1} = C$ match expects this new match to be with a $\pi = 1/2$ type and therefore expects output to be either $y = C$ or $y = B$, each with probability $1/2$. Producing $y = C$ leaves the new match in place, while producing $y = B$ leads to a separation. In either event, the low type's continuation payoff after the current period is $\beta C/(1-\beta)$. Therefore the low type does not gain from applying when his one-period expected income does not exceed C , the output in his current match:

$$\frac{1}{2}w_C + \frac{1}{2}w_B \leq C. \quad (4)$$

3. *Deters the high ($\pi = 1$) types from applying.*—Knowing that he is a high type, someone coming from a $y_{-1} = A$ match expects two possible outputs: $y = B$ or $y = A$, each with probability $1/2$. This type's initial expected wage must not exceed A , which is what he earns in his current high-output match:

$$\frac{w_B + w_A}{2} \leq A. \quad (5)$$

4. *Ensures that no firm profitably deviates to an AS-type match.*—A type $\pi = 1/2$ agent would learn his type immediately if matched for one period with a partner of known type. This is the Anderson-Smith (2010, “AS”) assignment, which sacrifices current output for an informational gain. Condition 4 requires that the separating equilibrium survives this deviation: no firm can profitably offer such a match. Denote by $J^{\pi, \pi'}$ the combined discounted lifetime payoff of both workers in a (π, π') -match. Thus J is a joint payoff. The corresponding payoff per worker is one half as large. We

first compare the gross joint payoff of the separating match with that of a one-period learning match with a known-low worker.

To show that $J^{1/2,1/2} > J^{0,1/2}$, note that matching a $\pi = 1/2$ agent with a $\pi = 0$ agent implies that

a) With probability $1/2$, joint output is $2C$ and both agents are low. The lifetime joint payoff is then $2C/(1 - \beta)$.

b) With probability $1/2$, joint output is $2B$ in the first period. In the best case, the newly discovered high type matches with another high type in the following period, so joint output is $A + C$ thereafter. The lifetime joint payoff is then $2B + \frac{\beta}{1-\beta}(A + C)$.

Thus the joint expected discounted output of agents in the “**0 and $\frac{1}{2}$ ” contract** is

$$J^{0,1/2} = \frac{1}{2} \left(\frac{2C}{1-\beta} \right) + \frac{1}{2} \left(2B + \frac{\beta}{1-\beta}(A + C) \right), \quad (6)$$

whereas for agents in the “ **$\frac{1}{2}$ and $\frac{1}{2}$ ” contract** it is

$$J^{1/2,1/2} = \frac{1}{4} \left(\frac{2C}{1-\beta} \right) + \frac{1}{4} \left(\frac{2A}{1-\beta} \right) + \frac{1}{2} (2B + \beta J^{1/2,1/2}). \quad (7)$$

Appendix 1 proves the following.⁵

Lemma 1 (*AS DEVIATION COMPARISON*) *If (2) holds, then for all $\beta \in (0, 1)$,*

$$J^{1/2,1/2} > J^{0,1/2}. \quad (8)$$

Intuitively, matching $\pi = 1/2$ types with $\pi = 0$ (known low) types gives full learning with probability 1 for the $\pi = 1/2$ type, but no learning for the $\pi = 0$ type. On the other hand, the match of two $\pi = 1/2$ types gives each member of the match full learning with probability $1/2$. Since $1/2 + 1/2 = 1$, the informational gain is the same in expected terms. However, the expected output of the 0 and $\frac{1}{2}$ match is lower than the expected output of the $\frac{1}{2}$ and $\frac{1}{2}$ match.

⁵We thank an anonymous referee for suggesting this argument in the strictly supermodular case.

The firm-deviation condition also includes the payments needed to attract the two workers. A firm that uses a known-low worker must give that worker $C/(1-\beta)$ and must give the unresolved worker his equilibrium application value W . By ex-ante zero profit and symmetry, the two workers together receive the joint payoff of the separating match, so $2W = J^{1/2,1/2}$. Hence $W = \frac{1}{2}J^{1/2,1/2}$, and equations (6) and (7) give

$$W + \frac{C}{1-\beta} - J^{0,1/2} = \frac{(1-\beta)(A+C-2B)}{2(2-\beta)} > 0.$$

Thus the AS-type match with a known-low worker is not profitable for a firm. The known-high case gives the same conclusion: relative to the known-low case, the required payment and the AS-type joint output both rise by $(A-C)/(1-\beta)$.⁶

Appendix 1a establishes existence of a separating equilibrium:

Proposition 1 (*SEPARATING EQUILIBRIUM*) Suppose (2) holds. For any $\phi \in (0,1)$ and $\beta \in (0,1)$, there exist (w_C, w_B, w_A) satisfying (3)–(5) and the ex-ante zero-profit condition $w_C + 2w_B + w_A = C + 2B + A$. A continuum of such contracts, indexed by δ , is

$$w_C = 0, \quad w_B = 2C - \delta, \quad w_A = A + 2B - 3C + 2\delta, \quad \delta \in [0, \min\{2C, A + C - 2B\}]. \quad (9)$$

Note that $w_C = 0$ in Eq. (9) does not mean a zero *lifetime* income for workers who produce $y = C$. After producing $y = C$, the pair remains together and earns output C per worker each period thereafter. The continuation payoff $\beta C/(1-\beta)$ is included on the left-hand side of condition (3). In that sense the firm retains the current period's low output while the workers retain their continuation value.

In equilibrium, agents sort positively on beliefs π . Since $y_{-1} \in \{C, A\}$ types are screened out by conditions (4) and (5), the shares of high and low types among

⁶If an AS-type deviation were profitable, it also would be feasible in the aggregate. In other words, experimenting with all $\pi = 1/2$ types would be feasible. The number of agents in low and high matches weakly exceeds the number in middle matches, and that is true for any ϕ :

$$(1-\phi)^2 + \phi^2 \geq 2\phi(1-\phi).$$

This policy would yield the AS outcome discussed in Section 4.2.

applicant pool participants each equal $1/2$, so ϕ does not enter the incentive constraints. The ex-ante zero-profit condition leaves two free wage components, and conditions (3)–(5) restrict them through inequalities. They therefore generally admit a continuum of equilibrium contracts.

Nevertheless, each wage schedule generates the same aggregate dynamics. Let $N_{y,t}$ be the share of agents in matches with output y at date t . Theorem 1 shows that the transition path and the convergence rate are identical across all separating equilibria.

Theorem 1 (*UNIQUENESS OF THE TRANSITION PATH*) *Let w and w' be any two separating-equilibrium wage schedules. The transition paths $\{N_{y,t}\}$ and $\{N'_{y,t}\}$ are then identical. Each path converges at the rate $1/2$ per period.*

Proof. Suppose, on the contrary, that w and w' generate different paths. The two paths start from the same random assignment, with belief-output pairs $(\pi, y) \in \{(0, C), (\frac{1}{2}, B), (1, A)\}$. Any difference in paths then requires different dissolution behavior at some date t . Consider the three match types in turn. Suppose that under w' some $y = B$ pairs remain in their current match instead of entering the applicant pool at date t . Those workers have $\pi = 1/2$, and zero profit together with (2) makes condition (3) strict. They therefore strictly prefer the applicant pool to staying, contradicting that w' is a separating equilibrium. Symmetrically, suppose some $y = C$ pair dissolves under w' : Those workers have $\pi = 0$, and condition (4) gives them no gain from dissolution. In a separating equilibrium they stay. Likewise for $y = A$ pairs by (5). The dissolution rule is therefore the same under every separating equilibrium: $y = B$ pairs always dissolve and re-enter the applicant pool, $y \in \{C, A\}$ pairs never do. Starting from the same initial conditions, this common rule implies the same law of motion under every separating equilibrium. At each date only workers in $y = B$ pairs enter the pool, and among them the fractions of high and low types are equal to $1/2$. Random matching therefore yields outputs C , B , and A with probabilities $1/4$, $1/2$, and $1/4$. Hence the next period's state must satisfy

$$N_{B,t+1} = \frac{1}{2}N_{B,t}, \quad N_{C,t+1} = N_{C,t} + \frac{1}{4}N_{B,t}, \quad N_{A,t+1} = N_{A,t} + \frac{1}{4}N_{B,t},$$

independently of the wage schedule. Therefore w and w' cannot generate different paths, a contradiction. ■

Present discounted value.—We next compare the present discounted value of output under the separating equilibrium with alternative output-contingent contracting equilibria.

Theorem 2 (*DISCOUNTED OUTPUT OPTIMALITY*) *Suppose (2) holds. The separating equilibrium of Proposition 1 maximizes the present discounted value of aggregate output among the output-contingent contracting equilibria with private histories considered here.*

Proof. *Applicants have identical histories.*—Under the separating equilibrium, an agent participates in the applicant pool at date t if and only if he has produced $y = B$ in every period since the shock. This follows directly from the equilibrium structure: Agents who produced $y \in \{C, A\}$ in any past period had their types revealed and left the pool permanently. Agents with $\pi \in \{0, 1\}$ are screened out by conditions (4) and (5). Therefore, every agent who enters the applicant pool at date t has the identical private output history $(B, B, \dots, B)$ of length t .

History-contingent contracts add no information.—Because all applicants at date t share the same history, a dynamic contract that conditions on an applicant's past output sequence cannot distinguish among them. History-conditioning at date t is therefore equivalent to conditioning on the date alone. The applicant pool is half high types at every date, so date-dependent wages face the same incentive constraints (3)–(5) at every t and produce the same equilibrium transition path. Thus history-contingent or date-dependent output-contingent wages do not improve on the present discounted value delivered by the static separating equilibrium.

Directed-learning alternatives.—Pairing known-low and known-high workers lowers output, since their combined output is $2B$ rather than $A + C$ under assortative matching. The remaining candidate improvement is to sacrifice current output to accelerate learning, as under AS. With two deterministic types, rematch only unresolved workers cannot reveal types faster than the separating path: a match of two

unresolved workers reveals both types only when output is C or A , and otherwise returns both workers to the same unresolved pool. Faster learning therefore requires matching some unresolved workers with known-type partners. The net-payoff comparison following Lemma 1 implies $J^{1/2,1/2} + 2C/(1 - \beta) > 2J^{0,1/2}$, with the analogous inequality for known-high partners. Since these comparisons apply to each pair of AS-type matches, assigning any positive mass of unresolved workers to known-type partners lowers aggregate discounted output. Therefore the separating equilibrium maximizes the present discounted value of aggregate output within the class of output-contingent contracting equilibria with private histories considered here. ■

The rate of convergence is therefore the same under every separating contract, and no alternative in this class raises discounted output. A broader characterization of mechanisms that use information beyond realized match output is outside the paper's scope.

4 Benchmarks

In this section, we compare the separating-contract path with two benchmarks. The public-history benchmark gives an upper bound on sorting speed, while the Anderson-Smith benchmark compares the speed and output cost of matching unresolved workers with known-type partners.

4.1 Public-history benchmark

Histories are private in the separating-contract equilibrium. We consider here the benchmark in which histories are public and reassignment can condition on past outputs.

Aggregate output is maximized by a positive assignment, and the matches with $y \in \{C, A\}$ are already composed optimally. It is the members of matches that produced $y = B$ that must re-match. With public histories this can be achieved in at most two periods. A firm could hire four workers from two matches that each produced output per worker $y = B$ and keep the four-worker set fixed. The group

contains two high types and two low types and can be paired in only three ways. Since the period-0 pairing generated outputs (B, B) , one of the two remaining pairings yields the positive assignment and output per worker of $(A + C)/2$.

The present value per two-worker match under this assignment is

$$J = 2 \left(\frac{1}{4} (A + C) + \frac{1}{2} B + \frac{\beta}{1 - \beta} \frac{A + C}{2} \right) > \frac{2B}{1 - \beta}.$$

However, the privacy of histories prevents a firm from coordinating four agents from two $y_{-1} = B$ matches, so that it is impossible to ensure that the group of four agents contains two low types and two high types. This benchmark serves two purposes. First, it provides an information-rich upper bound on sorting speed: Even a planner with full access to output histories can reach the positive assignment within two periods. It is therefore a benchmark against which the geometric contract path derived above can be compared.⁷ Second, it isolates what the initial shock costs regardless of the contracting setting: Both the public-history planner and the contracting equilibrium start from the same randomly assigned period-0 output distribution, so neither can avoid the transition costs of that random assignment. The two paths differ only in how quickly those costs are recovered.

Public histories are essential to this benchmark. A constrained planner who must instead respect the privacy of histories faces the same informational limitation as the market: he cannot identify individual types from team output $y = B$. In that case the contracting equilibrium of Section 3 is itself the relevant constrained benchmark within the output-contingent contracting environment studied here.

4.2 Anderson-Smith matching

The Anderson-Smith (2010) matching assignment, hereafter AS, provides a directed-learning benchmark. Under AS, a planner observes agents' private beliefs and directs matching on that basis. Each $\pi = 1/2$ agent is matched with a known-type ($\pi \in \{0, 1\}$) partner for one period. One period of production with a known-type partner

⁷At the first reassignment date, the high-high shares under this benchmark and the separating-contract path are both $\phi^2 + \phi(1 - \phi)/2$. Appendix 5 derives this and uses it in the proof of Corollary 1.

reveals the uncertain agent's type with probability one. If the partner is $\pi = 0$, the output is either B or C , each with probability $\frac{1}{2}$, and both outcomes identify the uncertain agent's type. If the partner is $\pi = 1$, the output is either A or B , each with probability $\frac{1}{2}$, and again both outcomes are identifying. Under the output function (1), any match with a known-type partner therefore delivers complete learning after one period.

Feasibility.—At the beginning of period 1, the worker population consists of $\pi = 0$ agents (fraction $(1 - \phi)^2$), $\pi = 1/2$ agents (fraction $2\phi(1 - \phi)$), and $\pi = 1$ agents (fraction ϕ^2). The total fraction of known-type agents is $(1 - \phi)^2 + \phi^2$, which weakly exceeds the fraction of uncertain agents $2\phi(1 - \phi)$ since $(1 - \phi)^2 + \phi^2 - 2\phi(1 - \phi) = (1 - 2\phi)^2 \geq 0$. Therefore each $\pi = 1/2$ agent can always be matched with a known-type partner, and no uncertain agent needs to match with another uncertain agent under AS. Among those partners, each uncertain agent is matched with a $\pi = 0$ partner with probability $(1 - \phi)^2 / [(1 - \phi)^2 + \phi^2]$ and with a $\pi = 1$ partner with the complementary probability.

Known types not used as partners are paired assortatively. Writing $K = (1 - \phi)^2 + \phi^2$ and $U = 2\phi(1 - \phi)$, the period-1 high-high mass is $N_{A,1}^{\text{AS}} = U\phi^2 / K + \phi^2(1 - U/K) = \phi^2$. The same calculation gives $N_{C,1}^{\text{AS}} = (1 - \phi)^2$, and hence aggregate output at date 1 equals initial aggregate output.

Information requirement and why AS is not a decentralized outcome. —The AS assignment requires that a planner observe agents' private beliefs and direct matching accordingly. In particular, the planner must distinguish $\pi = 1/2$ agents (who have always been in middle matches) from $\pi = 0$ and $\pi = 1$ agents. However, beliefs are derived from private output histories, and no firm or worker in the decentralized market can observe another agent's private history. Therefore, AS cannot arise as a competitive equilibrium under private histories. Instead, it serves as a benchmark for the speed of sorting that is achievable when private histories are set aside by assumption.

Why AS is dominated by the contract equilibrium.—Faster learning under AS comes at a cost in current output. A $\pi = 1/2$ agent matched with a $\pi = 0$ partner produces expected output $\frac{1}{2}(B + C)$, which is below the expected output B earned in

a middle match where both partners have $\pi = 1/2$. Lemma 1 in Section 3 establishes that the joint discounted return from matching two uncertain agents, $J^{1/2,1/2}$, strictly exceeds the return from matching an uncertain agent with a known-low agent, $J^{0,1/2}$, for all $\beta \in (0, 1)$ under (2). The same section also shows that, after paying the known worker and the unresolved worker their equilibrium values, the AS-type match is not profitable for a firm. Moreover, Appendix 5 shows that the contract equilibrium has a higher present discounted value of aggregate output than AS for all $\beta \in (0, 1)$, even though AS reaches the steady state in exactly two periods while the contract equilibrium approaches it asymptotically.

5 Equilibrium if contracts fail

We now consider the no-contract environment. Firms may be unable to commit to delivering output-contingent payments after output has been produced. Then contracts cannot screen unresolved agents from known-type agents, and meetings are governed by adverse selection. Following Smith (1992), output is divided equally within each match. Side payments are also excluded. An enforceable transfer would require the commitment technology assumed absent here, while a non-binding offer would reveal which partner benefits from preserving a mixed match. Appendix 4 gives the argument in detail.

Agents observe only their own output histories and decide whether to dissolve the current match. If a match dissolves, agents are immediately matched at random with new partners at the start of the period, and there is no unemployment. We refer to this rematching as recombination. A match persists unless at least one partner weakly prefers dissolution. Some dissolutions are therefore involuntary, because one partner prefers dissolution while the other prefers to keep the match.

Adverse selection in meetings.—While meetings continue, the meeting pool consists of agents who have produced $y \in \{C, B\}$. A match with $y = A$ is composed of high types and withdraws from the process. A match that produced C contains members who know that they are low types, but they can hope to raise their earnings

if they meet a high type, and so they separate voluntarily and enter the meeting pool. The only non-trivial decision rests with members of the middle matches. Since high-output matches never dissolve, an agent in a middle match could not have come from a high-output match. Thus, after the first recombination round, a middle match contains either two agents whose types are still unresolved, or one unresolved agent and one agent who knows he is a low type. In the second case, the known low type would like to keep the match, but the unresolved partner may prefer to dissolve it.

While recombination continues, all middle matches dissolve, and *all* the $1 - \phi$ low types recombine as long as the probability of meeting a high type is positive. Let n_t be the number of high types in the meeting pool at the start of period t , and let λ_t be the fraction of the meeting pool that is high. Then

$$\lambda_t = \frac{n_t}{n_t + 1 - \phi}, \quad (10)$$

with the initial condition

$$n_0 = \lambda_0 = \phi. \quad (11)$$

The date-0 value records the initial random assignment. The first voluntary recombination decision is evaluated at date 1, after period-0 output is observed and beliefs have been formed. When a high type matches with another high type, they produce A per worker and withdraw from the rematching process. A high type withdraws with probability λ_t . Therefore n_t satisfies the difference equation

$$n_{t+1} = (1 - \lambda_t) n_t, \quad (12)$$

with initial condition (11). Solving (10) and (12) gives

$$\lambda_{t+1} = \frac{\lambda_t}{1 + \lambda_t}, \quad \lambda_t = \frac{\phi}{1 + t\phi}, \quad (13)$$

and

$$n_t = \frac{\phi(1 - \phi)}{1 + (t - 1)\phi}, \quad (14)$$

which are valid while meetings continue. Thus the fraction of high types in the pool falls arithmetically.

Beliefs.—Let π_t be the beginning of period- t belief of an agent that he is a high type.⁸ The likelihood of producing $y = B$ conditional on being a high type is $1 - \lambda_t$, and the likelihood of producing $y = B$ conditional on being a low type is λ_t . Bayes rule then says

$$\pi_{t+1} = \frac{(1 - \lambda_t) \pi_t}{\lambda_t (1 - \pi_t) + (1 - \lambda_t) \pi_t} \quad \text{with } \pi_0 = \phi. \quad (15)$$

Appendix 3 shows that $\pi_1 = 1/2$ and that for $t \geq 2$,

$$\pi_t = \left(1 + \prod_{i=1}^{t-1} \frac{\lambda_i}{1 - \lambda_i} \right)^{-1}. \quad (16)$$

The recombination decision.—The expected output from recombining depends on the agent's beginning-of-period belief π and on the fraction λ of high types in the meeting pool:

$$\begin{aligned} v(\pi, \lambda) &= \pi(\lambda A + (1 - \lambda)B) + (1 - \pi)(\lambda B + (1 - \lambda)C) \\ &= B + (B - C)[\pi\lambda\rho - (1 - \pi)(1 - \lambda)], \end{aligned} \quad (17)$$

where

$$\rho \equiv \frac{A - B}{B - C} > 1. \quad (18)$$

The inequality follows from strict supermodularity. Suppose first that period t is the last one in which recombination could occur. Staying with the current middle match gives $B/(1 - \beta)$. Recombining once and then keeping the new match gives $v(\pi_t, \lambda_t)/(1 - \beta)$. Thus the discount factor cancels, and a member of a middle match recombinates when $v(\pi_t, \lambda_t) \geq B$. Define the reservation belief $\hat{\pi}(\lambda)$ by

$$\hat{\pi}(\lambda) = \frac{1 - \lambda}{\rho\lambda + 1 - \lambda}. \quad (19)$$

⁸Doppelt (2016), who also has two types, refers to π_t as the worker's résumé.

The one-period condition is equivalently $\pi_t \geq \hat{\pi}(\lambda_t)$. At earlier dates, recombining also preserves the possibility of learning from a new middle match. Let V_t be the value of recombining forever at date t for an agent who has always been in a middle match. Recombining forever is incentive compatible if and only if the following condition holds:

$$V_t \geq \frac{B}{1-\beta} \quad \text{for all } t \geq 1. \quad (\text{IC})$$

Maximal-duration recombination equilibrium.—A recombination round is incentive compatible if and only if an unresolved agent in a middle match weakly prefers dissolution given that all other agents who would participate in that round also do so. When several paths satisfy these incentives, we take the one with the largest number of recombination rounds. Recombination decisions are first made at the beginning of period 1, after period-0 output is observed and beliefs are formed. Let $T \geq 0$ be the number of recombination rounds in the maximal-duration equilibrium. Since the initial period is date $t = 0$, $T = 0$ means that agents do not recombine at the first opportunity, which is the beginning of period 1.

Appendix 2 proves the following.

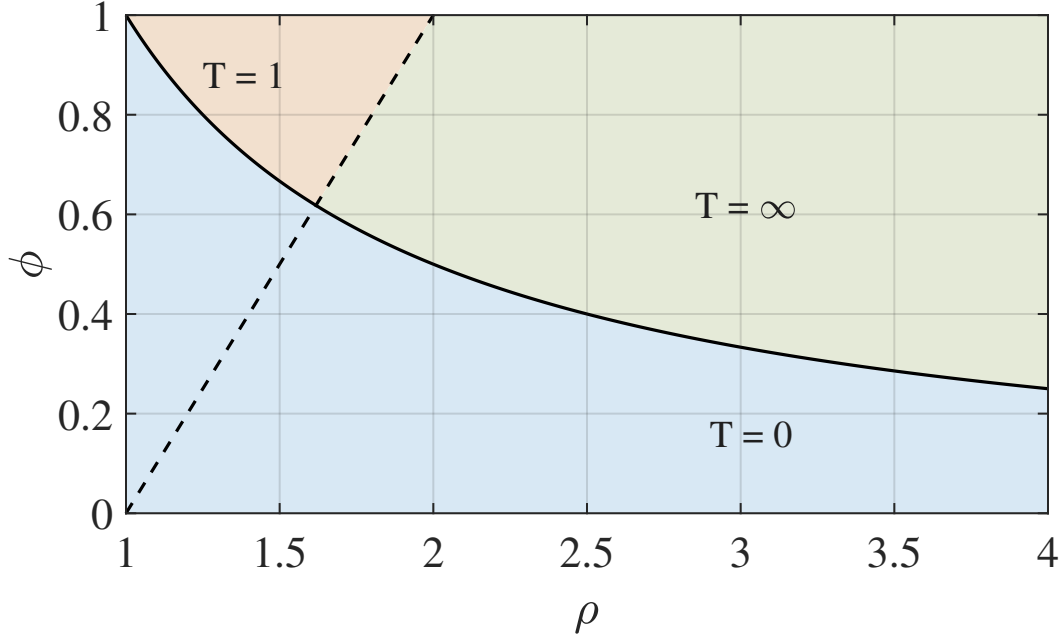
Proposition 2 (*THREE TYPES OF EQUILIBRIUM IN THE NO-CONTRACT CASE*) *Let ρ be defined by (18).*

- (i) *If $\rho\phi < 1$ and (IC) fails,⁹ then $T = 0$, i.e., no recombination.*
- (ii) *If $\rho\phi \geq 1$, $\rho < 1 + \phi$, and (IC) fails, then $T = 1$, i.e., one period of recombination.*
- (iii) *In all other cases, $T = \infty$, i.e., recombination forever.*

Figure 1 illustrates the three regions in Proposition 2. Below $\rho\phi = 1$, recombining once and then keeping the new match is not profitable. When $\rho\phi \geq 1$, the boundary $\rho = 1 + \phi$ gives the second comparison.

⁹Condition (IC) can equivalently be written as a lower bound on β . Appendix 2 derives $\beta^*(\rho, \phi)$ and shows that the condition holds if and only if $\beta \geq \beta^*(\rho, \phi)$. A high type who recombines forever eventually enters a high-high match because $\sum_{s=t}^{\infty} \lambda_s = \infty$, while a low type's expected output converges to C as $\lambda_s \rightarrow 0$. Since $\pi_t \geq 1/2$, the limiting average payoff is at least $(A + C)/2 > B$ by (2). Since the one-period threshold for ρ converges to zero, only finitely many dates can bind. Thus (IC) holds when β is sufficiently close to one, and finite recombination requires sufficiently impatient agents.

Figure 1: Recombination regions as a function of (ρ, ϕ)



Notes: The boundaries $\rho\phi = 1$ and $\rho = 1 + \phi$ divide the one-period comparisons at the first and second recombination dates. The labels $T = 0$ and $T = 1$ apply when (IC) fails. If (IC) holds, then $T = \infty$. The upper-right region satisfies (IC) for every $\beta \in (0, 1)$, where $\rho = (A - B)/(B - C)$.

If $\rho\phi \geq 1$ and $\rho \geq 1 + \phi$, the one-period condition holds at every date, so recombination continues forever. In the other two regions, sufficiently patient agents recombine forever, while sufficiently impatient agents stop after zero or one round as described in Proposition 2.

When recombination continues indefinitely, the equilibrium shares of agents in the three match types under the no-contract regime, denoted NC, are

$$\begin{aligned}
 N_{C,t}^{\text{NC}} &= (1 - \phi + n_t) (1 - \lambda_t)^2 \rightarrow 1 - \phi \\
 N_{B,t}^{\text{NC}} &= (1 - \phi + n_t) \{1 - (1 - \lambda_t)^2 - \lambda_t^2\} \rightarrow 0 \\
 N_{A,t}^{\text{NC}} &= \sum_{j=0}^t (1 - \phi + n_j) \lambda_j^2 \rightarrow \phi.
 \end{aligned} \tag{20}$$

If $T = 1$, then for $t \geq 1$ the distribution across match types stabilizes at the fractions

$$N_{C,t}^{\text{NC}} = \frac{1-\phi}{1+\phi}, \quad N_{B,t}^{\text{NC}} = \frac{2\phi(1-\phi)}{1+\phi}, \quad N_{A,t}^{\text{NC}} = \frac{2\phi^2}{1+\phi}. \quad (21)$$

Under separating contracts, the middle share halves each period:

$$N_{B,t}^{\text{C}} = 2\phi(1-\phi) \left(\frac{1}{2}\right)^t. \quad (22)$$

Without contracts, the high share in the meeting pool is $\lambda_t = \phi/(1+t\phi)$ while meetings continue. Thus contracts make the transition geometric. Equal splitting and adverse selection make it arithmetic and may stop it altogether.

6 Distributional implications

The paper's main result is about the speed of sorting. The same transition also has a distributional implication: As mixed matches disappear, mass moves out of the middle output state. This section states that implication using the increasing convex order.

Let F_t^i denote the date- t distribution of match output per worker in regime $i \in \{\text{PH}, \text{C}, \text{AS}, \text{NC}\}$, supported on $\{C, B, A\}$. The common initial distribution is denoted F_0 . Here PH denotes the public-history benchmark and C the separating-contract regime. Recall that $F \succeq_{icx} G$ in the *increasing convex order* if $\int u dF \geq \int u dG$ for every increasing convex u . Equivalently, $E_F[(Y-k)^+] \geq E_G[(Y-k)^+]$ for every k (Shaked and Shanthikumar 2007).

In every regime each match is low-low, high-low, or high-high. Let $s_t^i \equiv N_{A,t}^i$ be the share of agents in high-high matches. Since $\phi = s_t^i + N_{B,t}^i/2$, $N_{B,t}^i = 2(\phi - s_t^i)$ and $N_{C,t}^i = 1 - 2\phi + s_t^i$. Mean output is therefore

$$Y_t^i = C + 2\phi(B - C) + s_t^i(A - 2B + C). \quad (23)$$

Since $A - 2B + C > 0$, the mean and the top share move together.

Appendix 5 proves the following.

Corollary 1 (*INCREASING-CONVEX-ORDER RANKING*)

- (i) *In every regime that adjusts, the distribution of match output rises over time in the increasing convex order, and $F_t^i \succeq_{icx} F_0$ for all $t \geq 1$.*
- (ii) *At each date $t \geq 1$, $F_t^{\text{PH}} \succeq_{icx} F_t^{\text{C}} \succeq_{icx} F_t^{\text{NC}} \succeq_{icx} F_0$, with weak inequalities.*
- (iii) *The Anderson-Smith and contract distributions cross once: $F_1^{\text{C}} \succeq_{icx} F_1^{\text{AS}}$ and $F_t^{\text{AS}} \succeq_{icx} F_t^{\text{C}}$ for all $t \geq 2$.*

Part (ii) ranks the public-history, contract, and no-contract distributions against the initial one at every date. Part (iii) shows that the directed-learning benchmark, which gives up output early to reveal types faster, is dominated by the contract distribution at $t = 1$ but dominates it for all $t \geq 2$.

7 Conclusion

This paper characterizes how adverse selection slows an economy's adjustment to a major technological shock that garbles agents' abilities and forces agents to relearn their types. When separating contracts are available, adjustment is geometric: The middle share halves each period. When contracts fail and output is split equally, known low types re-enter the meeting pool, the fraction of high types in that pool falls arithmetically, and for some parameter values adjustment stops altogether. Thus the speed of convergence to assortative matching depends on whether firms can commit to output-contingent wage schedules that screen workers by their private histories.

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Appendix

Learning and Assortative Matching with Private Histories

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Appendix 1: Proof of Lemma 1

Simplifying (6) yields

$$J^{0,1/2} = B + \frac{C + \frac{1}{2}\beta(A + C)}{1 - \beta}.$$

Simplifying (7) yields

$$J^{1/2,1/2} = \frac{A + C + 2(1 - \beta)B}{(1 - \beta)(2 - \beta)}.$$

Therefore

$$J^{1/2,1/2} - J^{0,1/2} = \frac{g(\beta)}{2(1 - \beta)(2 - \beta)},$$

where

$$g(\beta) = [2 - \beta(2 - \beta)]A + 2\beta(1 - \beta)B - (2 - \beta^2)C.$$

Now

$$g'(\beta) = 2[-A + B + \beta(A - 2B + C)], \quad g''(\beta) = 2(A - 2B + C) > 0.$$

The zero of g' is

$$\frac{A - B}{A - 2B + C} > 1,$$

since $B > C$. Thus g is decreasing on $[0, 1]$. Since $g(1) = A - C > 0$, it follows that $g(\beta) > 0$ for all $\beta \in (0, 1)$. Hence $J^{1/2,1/2} > J^{0,1/2}$. $\square$

Appendix 1a: Proof of Proposition 1

Proof of Proposition 1. Ex-ante zero profit holds since

$$w_C + 2w_B + w_A = 0 + 2(2C - \delta) + A + 2B - 3C + 2\delta = A + 2B + C.$$

Condition (4) requires $(2C - \delta)/2 \leq C$, which holds for $\delta \geq 0$. Condition (5) requires

$$\frac{2C - \delta + A + 2B - 3C + 2\delta}{2} \leq A,$$

or $\delta \leq A + C - 2B$, which holds on the stated interval. To verify (3), observe that δ cancels in W :

$$W = \frac{(1 - \beta)(A + 2B + C) + \beta(A + C)}{2(1 - \beta)(2 - \beta)}.$$

Then $W \geq B/(1 - \beta)$ if and only if

$$(1 - \beta)(A + 2B + C) + \beta(A + C) \geq 2B(2 - \beta),$$

which reduces to $A + C \geq 2B$. The inequality is strict by (2), so (3) holds strictly. $\square$

Appendix 2: Proof of Proposition 2

Proof of Proposition 2.—Let L_t and V_t be the values of recombining forever for a known low type and for an agent who has always been in a middle match. Since payoffs are bounded and $\beta < 1$, these values are the unique bounded solutions to

$$\begin{aligned} L_t &= \lambda_t B + (1 - \lambda_t)C + \beta L_{t+1}, \\ V_t &= \pi_t \lambda_t \frac{A}{1 - \beta} + (1 - \pi_t)(1 - \lambda_t)(C + \beta L_{t+1}) \\ &\quad + [\pi_t(1 - \lambda_t) + (1 - \pi_t)\lambda_t](B + \beta V_{t+1}). \end{aligned}$$

Because a known low type in a $y = C$ match prefers to recombine whenever $\lambda_t > 0$, recombination forever is incentive compatible if and only if (IC) holds.

At the first recombination date,

$$\lambda_1 = \frac{\phi}{1 + \phi} \quad \text{and} \quad \pi_1 = \frac{1}{2}.$$

Using (17),

$$v(\pi_1, \lambda_1) - B = \frac{B - C}{2(1 + \phi)}(\rho\phi - 1).$$

Thus the first-date condition is $\rho\phi \geq 1$.

At the second recombination date,

$$\lambda_2 = \frac{\phi}{1+2\phi} \quad \text{and} \quad \pi_2 = \frac{1}{1+\phi}.$$

Evaluating (17) at $t = 2$ gives

$$v(\pi_2, \lambda_2) - B = \frac{\phi(B-C)}{(1+\phi)(1+2\phi)} (\rho - 1 - \phi).$$

Thus the second-date condition is $\rho \geq 1 + \phi$.

To characterize (IC), put $a = 1/\phi$, $b_t = a + t - 1$, $p_t = (1 - \pi_t)/\pi_t$, and $\Psi_b(\beta) = (1 - \beta) \sum_{k \geq 0} \beta^k / (b + 1 + k)$. Conditioning on type shows that the date- t inequality in (IC) is equivalent to $D_t(\beta) \equiv \rho - p_t - (\rho b_t - p_t) \Psi_{b_t}(\beta) \geq 0$. Here $p_1 = 1$, $p_2 = \phi$, and $p_{t+1} = p_t/b_t$. Since $\rho b_t > p_t$ and Ψ_b decreases from $1/(b+1)$ to zero, D_t is increasing in β . The one-period condition is $\rho \geq q_t = p_t b_t$, where $q_2 = 1 + \phi$ and $q_{t+1} \leq q_t$ for $t \geq 2$. Put $x_k = b_t + 1 + k$. Direct substitution shows that $D_{t+1} - D_t/b_t$ is a sum whose k th term has positive weight $(1 - \beta)\beta^k / [b_t x_k (x_k + 1)]$ and numerator $\rho[(b_t - 1)x_k(x_k + 1) - b_t^2 x_k + b_t] - p_t$. For $t \geq 2$, since $b_t > 2$ and $0 < p_t \leq 1$, this numerator starts at $\rho(b_t^2 - 2) - p_t > 0$ and rises by $\rho[b_t^2 + 2b_t - 4 + 2(b_t - 1)k] > 0$. Hence $D_t \geq 0$ implies $D_{t+1} > 0$.

Set $\beta_1^* = 0$ if $\rho\phi \geq 1$ and otherwise define it by $\Psi_a(\beta_1^*) = (\rho - 1)/(\rho a - 1)$. Set $\beta_2^* = 0$ if $\rho \geq 1 + \phi$ and otherwise define it by $\Psi_{a+1}(\beta_2^*) = (\rho - \phi)/(\rho(a + 1) - \phi)$. Each positive cutoff is unique. Thus (IC) is equivalent to $\beta \geq \beta^*(\rho, \phi)$, where $\beta^* = \max\{\beta_1^*, \beta_2^*\}$.

When $\beta < \beta^*(\rho, \phi)$, (IC) fails. Since q_t declines for $t \geq 2$, any last round $T \geq 2$ would permit all later rounds and imply (IC), a contradiction. Thus the date-1 and date-2 conditions prove parts (i) and (ii). Otherwise (IC) holds and maximal duration gives $T = \infty$, proving part (iii), with $\beta^* = 0$ when $\rho\phi \geq 1$ and $\rho \geq 1 + \phi$. $\square$

Derivation of Eqs. (20) and (21).—At date t the total number of agents in the pool is $1 - \phi + n_t$. Among these pool members, a share $(1 - \lambda_t)^2$ end up in low-low matches, a share $1 - (1 - \lambda_t)^2 - \lambda_t^2$ end up in middle matches, and a share λ_t^2 end up in high-high

matches. The latter are added to the stock of previous high-high matches, giving (20). For $T = 1$, using $n_1 = \phi(1 - \phi)$ and $\lambda_1 = \phi/(1 + \phi)$ gives (21).

Appendix 3: Derivation of Eq. (16)

We solve for π_t given the λ_t sequence. Dividing top and bottom by π_t , (15) reads

$$\pi_{t+1} = \frac{1 - \lambda_t}{\lambda_t \frac{1 - \pi_t}{\pi_t} + 1 - \lambda_t}.$$

Now let $m_t = \frac{1 - \pi_t}{\pi_t}$ so that $\pi_t = 1/(1 + m_t)$. Then

$$\lambda_t m_t = (1 - \lambda_t) m_{t+1}.$$

Therefore

$$m_t = \left(\prod_{i=0}^{t-1} \frac{\lambda_i}{1 - \lambda_i} \right) m_0.$$

Finally,

$$\begin{aligned} \pi_t &= \left(1 + \prod_{i=0}^{t-1} \frac{\lambda_i}{1 - \lambda_i} \frac{1 - \pi_0}{\pi_0} \right)^{-1} \\ &= \left(1 + \prod_{i=1}^{t-1} \frac{\lambda_i}{1 - \lambda_i} \right)^{-1} \text{ for } t \geq 2, \end{aligned}$$

because $\pi_0 = \lambda_0 = \phi$. For $t = 1$,

$$\pi_1 = \left(1 + \frac{\lambda_0}{1 - \lambda_0} \frac{1 - \pi_0}{\pi_0} \right)^{-1} = \frac{1}{2}.$$

Appendix 4: Side Payments in the No-Contract Case

In a $y = B$ match, either both agents are uncertain or one is known low and the other uncertain. A side payment can matter only in the latter case: The known low type would like to keep the match, while the uncertain agent may prefer recombination.

Enforceable side payments.—An enforceable transfer from the known low type to the uncertain agent is a contract contingent on continuation of the match. That is precisely the commitment technology assumed absent in the no-contract environment. If such transfers could be enforced, firms or agents could also use output-contingent contracts to screen histories, returning the economy to the contract environment of Proposition 1.

Non-binding offers.—Suppose instead that the low type can make a non-binding offer to share some future output if the match stays together. The offer itself reveals the low type's private information. The uncertain agent then learns that his partner is known low and therefore that he is high. Recombining once gives expected output $\lambda_t A + (1 - \lambda_t)B > B$ whenever $\lambda_t > 0$, so he prefers to dissolve the match. Whether the promised transfer is paid later cannot affect the current dissolution decision unless the transfer is enforceable.

Bargaining.—One could instead model bargaining as in Golosov, Lorenzoni, and Tsyvinski (2014) or Wilson (2001). Such bargaining would determine how output is divided inside an existing match, but it would not by itself solve the private-history screening problem. In the no-contract case, convergence slows because known low types cannot be excluded from the pool while high-high pairs exit irreversibly. Allowing side payments that do not create enforceable history-contingent contracts does not change this.

Appendix 5: Increasing Convex Order and Discounted Output

Lemma 2 *On the support $\{C, B, A\}$, $F \succeq_{icx} G$ if and only if $E_F[Y] \geq E_G[Y]$ and $N_A^F \geq N_A^G$.*

Proof. The stop-loss transform $\Pi_F(k) = E_F[(Y - k)^+]$ is continuous, convex, and piecewise linear in k , with kinks only at C, B, A and $\Pi_F(k) = 0$ for $k \geq A$. Hence $\Pi_F \geq \Pi_G$ everywhere if and only if it holds for $k \leq C$ and at $k = B$. Since the support lies weakly above C , $(Y - k)^+ = Y - k$ for $k \leq C$, so there $\Pi_F(k) - \Pi_G(k) = E_F[Y] - E_G[Y]$. At $k = B$, $\Pi_F(B) = N_A^F(A - B)$, so the comparison at B is

equivalent to $N_A^F \geq N_A^G$. The intervals $[C, B]$ and $[B, A]$ are linear with endpoints already compared, so both segments follow. ■

Proof of Corollary 1.—For the distributions in the model, mean output is the affine function (23) of $s_t^i = N_{A,t}^i$. Since $A - 2B + C > 0$, Lemma 2 implies that the distributions are ordered by the top share. In every regime that adjusts, s_t^i is nondecreasing in t , with $s_0^i = \phi^2$, which proves part (i).

For $F_t^{\text{PH}} \succeq_{icx} F_t^{\text{C}}$, compare s_t . In the public-history benchmark, two $y = B$ matches form a four-worker group with two high and two low types. Of the three ways to pair them, one was used in period 0, and exactly one of the remaining two is positive. Since the previous B outputs do not reveal individual types, the first new pairing is positive with probability $1/2$. Hence $s_1^{\text{PH}} = s_1^{\text{C}} = \phi^2 + \phi(1 - \phi)/2$. If the first new pairing is not positive, the other one is, so $s_t^{\text{PH}} = \phi \geq s_t^{\text{C}}$ for $t \geq 2$. Lemma 2 then gives $F_t^{\text{PH}} \succeq_{icx} F_t^{\text{C}}$.

For $F_t^{\text{C}} \succeq_{icx} F_t^{\text{NC}}$, let $U_t^i = \phi - s_t^i$ be the mass of high types not yet in a high-high match. Under contracts, $U_{t+1}^{\text{C}} = \frac{1}{2}U_t^{\text{C}}$. Under no contracts a high type leaves this stock only by meeting another high type in the pool. At the first recombination date and thereafter, the high fraction in the meeting pool is $\lambda_t \leq \lambda_1 = \phi/(1 + \phi) \leq \frac{1}{2}$ by Eq. (13), and it is zero once recombination stops. Thus $U_{t+1}^{\text{NC}} \geq \frac{1}{2}U_t^{\text{NC}}$, and induction gives $s_t^{\text{NC}} \leq s_t^{\text{C}}$. Also $s_t^{\text{NC}} \geq \phi^2 = s_0$ whenever recombination occurs, with equality if it does not. This proves part (ii).

For part (iii), under AS, $Y_1^{\text{AS}} = Y_0$, so $s_1^{\text{AS}} = \phi^2 < \phi(1 + \phi)/2 = s_1^{\text{C}}$. AS attains the positive assignment at $t = 2$, so $s_t^{\text{AS}} = \phi \geq s_t^{\text{C}}$ for $t \geq 2$. Lemma 2 then yields $F_1^{\text{C}} \succeq_{icx} F_1^{\text{AS}}$ and $F_t^{\text{AS}} \succeq_{icx} F_t^{\text{C}}$ for $t \geq 2$. □

Discounted output comparison with AS.—Let $\Delta = \phi(1 - \phi)(A - 2B + C)$ and $Y_\infty = \phi A + (1 - \phi)C$. The common initial output is $Y_0 = Y_\infty - \Delta$, $Y_t^{\text{C}} = Y_\infty - \Delta(1/2)^t$ for $t \geq 0$, and the AS benchmark has $Y_0^{\text{AS}} = Y_0$, $Y_1^{\text{AS}} = Y_0$, and $Y_t^{\text{AS}} = Y_\infty$ for $t \geq 2$, so

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t (Y_t^{\text{C}} - Y_t^{\text{AS}}) &= \Delta \left[\frac{\beta}{2} - \sum_{t=2}^{\infty} \left(\frac{\beta}{2} \right)^t \right] \\ &= \Delta \frac{\beta(1 - \beta)}{2 - \beta} > 0. \end{aligned}$$